Dynamic Mass Density and Acoustic Metamaterials

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Abstract: Dynamic mass density of a composite can differ from its static counterpart when there are relative motions between the components. Realizations of negative dynamic mass density composites and their theoretical underpinnings will be presented. The issue of liquid-solid composite mass density at the low frequency limit will also be addressed.

Dynamic mass density may be defined as $\langle f \rangle / \langle a \rangle$, where $\langle f \rangle$ denotes the spatial averaged force density and $\langle a \rangle$ the averaged acceleration. In a multicomponent composite, if there can be *relative motions between the components*, then it is entirely possible to have dynamic mass density that differs from its static counterpart. A well-known example is the locally-resonant sonic materials [1], where the structural unit consists of a heavy spherical mass wrapped in a soft rubber coating. Since the structural units have an intrinsic resonance, it is not surprising that at frequencies higher than the resonance frequency, the heavy core particle will move *against* the matrix displacement of the incident wave. If the density of these structural units is sufficiently high, then the spatially-averaged mass density can be negative at such frequencies.

Negative mass density (NMD) metamaterials can be useful in attenuating low frequency sound, which has very high penetrating power through solid walls because of the mass density law that states the sound transmission amplitude $T \propto \left(\rho\omega t\right)^{-1}$, where t denotes the wall thickness. By moving out of phase with the incident wave, NMD metamaterials can exponentially attenuate the low frequency sound within its effective frequency range, thus breaking the mass density law. In the ultimate limit of such materials it would be desirable to have a thin and light-weight membrane that can operate effectively in the 100 - 1,000 Hz range. However, stopping low frequency sound with a thin membrane is against simple intuition, as total reflection requires the formation of a *node* (i.e., no motion) at the reflecting surface, and a membrane with weak elastic restoring force is an unlikely candidate to be a low frequency sound reflector.

We show that precisely because of the weak elastic moduli of the membrane, there can be various low-frequency oscillation patterns even within a small and finite sample with fixed boundaries as defined by a rigid grid. Such vibrational eigenmodes can be tuned by placing a small mass at the center of the membrane sample, and near-total reflection is achieved at a frequency in-between two eigenmodes where the in-plane *average displacement* (normal to the membrane) is zero, leading to exponentially small far-field transmission [2].

The fact that the effective mass density can be negative at finite frequencies does not answer the question about whether the dynamic mass density always has to equal its static counterpart, i.e., the volume average value, in the limit of frequency $\omega \rightarrow 0$. Mathematically, this is equivalent to asking the question about homogenization of the elastic wave operator/equation

$$\nabla \cdot \mu \left[\nabla \vec{u} + (\nabla \vec{u})^T \right] + \nabla (\lambda \nabla \cdot \vec{u}) + D\omega^2 \vec{u} = 0,$$

where $\vec{u}$ denotes displacement, μ the shear modulus, λ the longitudinal Lame constant and D the dynamic mass density. It can be seen that usually the homogenization is done by taking the limit of $\omega \to 0$ first, so that the homogenization is just on the Laplace operator only. We have shown that for a composite with a fluid matrix [3,4], one can obtain a different limiting expression (compared to the static volume average value) for the effective mass by letting $\mu / \rho\omega\ell^2 \to 0$ first, where ρ denotes the static mass density and ℓ the typical feature size in the composite. However, $D = \rho$ as $\omega \to 0$ for the solid matrix [4]. The difference in the static and dynamic mass densities is shown to explain some puzzling experimental results [3].

References

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