Frequency conversion cascade using non commutative space and time interfaces

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Abstract

Time varying meta-materials are a promising tool to manipulate waves. In particular, they offers a new way to perform frequency conversion without using nonlinearity or the Doppler effect. Their experimental implementation is nevertheless challenging as changing the propagation properties of a medium by a significant amount on a short time scale is difficult. A time interface consisting for instance of a temporal variation of the refractive index generates a frequency shift, the wave vector remaining unaltered. This is the temporal counterpart of the standard spatial interface where the wave vector varies while the frequency is kept constant. However, the amplitude of the refractive index variation in time is usually very limited, resulting in a small frequency shift. Here we propose to use space and time interfaces to perform iterative elementary frequency conversion and achieve arbitrary frequency conversion cascades. The elementary step is obtained by using an intermediate medium in which a wave packet of finite size enters and exits by a different interface, one being a time interface, the other a space interface. The two operations are not commutable, the time interface performs a small frequency conversion while the space interface shifts the wave vector. Thus, the wave packet after an elementary step has changed frequency and wave vector, propagating at the same initial speed in the absence of dispersion. Such a step transformation can be iterated to produce a frequency conversion cascade of arbitrary amplitude. Depending on the order of the crossing of the two interfaces, one obtains an upward or downward conversion. An elegant geometric interpretation of this cascade can be given using projection operations on the dispersion cone. The energy efficiency of the conversion process is given by the ratio of the output frequency to the input one and is found to be independent of the impedance mismatch between the two media.

We implement this frequency conversion cascade with electrostriction controlled water waves. An electrode is mounted parallel to the water surface. When an electric field is applied (typically 15 kV/cm), it generates a force on the conductive liquid which modifies the wave velocity under the electrode. Wave velocity can be reduced by up to a factor 1.5 compared to free water with a great control both in space and time. To perform a frequency conversion cascade, multiple electrodes are placed along the path of a propagating wave packet. By changing their voltages separately as the wave propagates under each electrode, frequency shifts that could not be reached with a single interface can be achieved. Without damping, frequency conversion cascades up to 4 octaves can be performed.

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