Classical and wave chaos in the transformation from curved to flat surfaces

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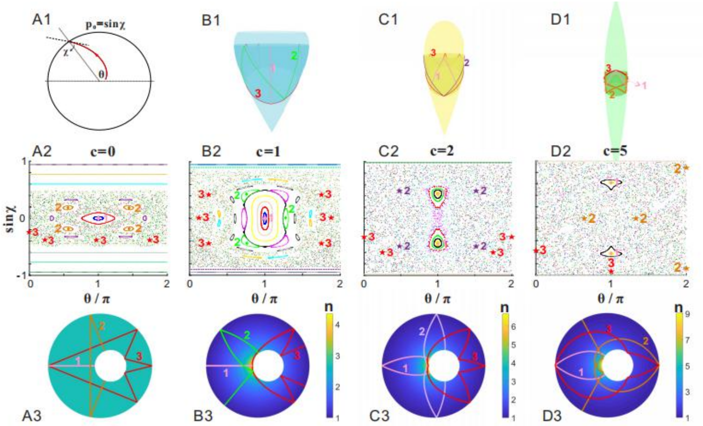
Light propagation on a two-dimensional curved surface embedded in a three-dimensional space has attracted increasing attention as an analog model of four-dimensional curved spacetime in laboratory. Nevertheless, investigation of nonlinear dynamics in non-Euclidean geometry is still scarce and remains challenging, owing to the absence of a power tool. Here, inspired by the concept of transformation optics, we illustrate that a curved surface is fundamentally equivalent to a table billiard with nonuniform distribution of refractive index, in terms of both light rays and waves. We study classical and wave chaotic dynamics of light on a specific family of surface of revolution and its equivalent conformally transformed flat billiard, when an off-centered hole is pierced to introduce chaos. We find that the degree of chaos is fully controlled by the single geometric parameter of the curved surface. This is proved by exploring in the transformed billiard the dependence with this geometric parameter of the Poincaré surface of section, the Lyapunov exponent and the statistics of eigenmodes and eigenfrequency spectrum. Finally, a simple interpretation of our findings naturally emerges when considering transformed billiards, which allows to extend our prediction to other class of curved surfaces. The powerful analogy we reveal here between two a priori unrelated systems not only brings forward a novel approach to control the degree of chaos, but also provides potentialities for further studies and applications in various fields, such as billiards design, optical fibers, or laser microcavities.

As an application, we present that by introducing spatially varying refractive index, it is possible to undo deterministic chaos in arbitrary two-dimensional optical chaotic billiards. The landscape of refractive index is obtained by a conformal transformation from an integrable billiard. We show that this approach is robust to small fluctuations. More interestingly, trajectory rectification can be realized by relating chaotic billiards with non-Euclidean billiards. Finally, we illustrate the universality of this approach by extending our investigations to arbitrarily deformed optical billiards.



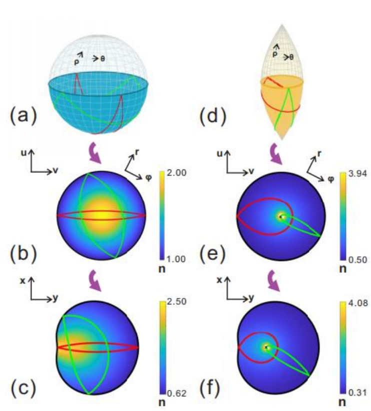


Fig. 1. Poincaré surfaces of section of Tannery's pears with different parameter c.

Fig. 2. Refractive index landscape in transformed Robnik billiards transformed from non-Euclidean billiards.