Invariance property of the Fisher information in scattering media

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We prove analytically and confirm numerically that the mean precision of measurements, averaged over input angles, frequency and configurations, is invariant with respect to the scattering strength of the environment surrounding the particle.

Precisely estimating the properties of sub-wavelength particles surrounded by scattering environments is a central aspect of many research areas, ranging from the measurement of the position and the mass of biological molecules [1, 2] to the characterization of engineered nanostructured samples [3]. In these contexts, multiple scattering effects are usually seen as a major drawback, limiting the achievable precision in the estimation of observable parameters characterizing the particles. Nevertheless, it is also known that such multiple scattering effects can sometimes be beneficial to optical imaging [4], especially if prior knowledge on the scattering environment is available to the observer [5]. These insights naturally raise the question of how the presence of a scattering environment around a sub-wavelength particle influences the information carried by the field scattered by the particle.

Here, we uncover that the measurement precision achievable in scattering systems is subject to a counterintuitive invariance rule: the scattering environment surrounding a sub-wavelength particle does, on average, *not* influence *at all* the precision that can be achieved when estimating the properties of this particle. Moreoever, the intimate connection between measurement and backaction allows us to interpret this result also as an invariance for the average micro-manipulation capabilites of waves in a scattering environment.

This insight is made possible through the Fisher information operator, that quantifies for an arbitrary incoming light state the quantum Fisher information (QFI) associated to a measurement [6]. The concept of QFI has enabled considerable progress in the precise characterization of sub-wavelength particles through optical measurements-[7, 8]. Specifically, we show that, in the canonical case of a flux-conserving system illuminated by a coherent field, the average QFI and the average quantum Cramér-Rao bound (QCRB) are both independent of the scattering strength of the environment surrounding the particle. Originating from a fundamental connection between the QFI, the local density of states (LDOS) and the cross density of states [9], these results show that a simple fundamental law rules the ultimate precision limit achievable in estimating properties of a sub-wavelength particle using coherent light scattering, due to the isotropic and homogeneous distribution of the average LDOS [10]. This invariance law holds both in the ballistic and in the diffusive regime, whereas deviations are observed when Anderson localization sets in. In this case, the average QCRB increases with the scattering strength of the medium. Remarkably, however, the average QFI increases in this case, as is explained by studying the influence of C_0–speckle correlations [11] on the average QFI. In addition, we quantify the increase of the average QFI obtained when the incident field is spatially optimized using wavefront shaping techniques.

We confirm these results using numerical simulations of a rectangular multi-mode waveguide. The computational results agree very well with the theoretical predictions. We note that the concepts used in the derivation are general, so this invariance property should hold in any system described by a linear wave equation.

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