



Time-reversal lens and phase conjugation with an active spatiotemporal interface [Invited]

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Abstract: Active spatiotemporal interfaces enable wave control through frequency conversion, phase conjugation, and negative refraction. Thin time-modulated interfaces have been predicted to act as flat time-reversal lenses, forming phase-conjugated images on the far side of the interface, yet experiments have mostly addressed phase conjugation or negative refraction in simple beam geometries rather than imaging. Here we realize a flat time-reversal lens for water waves using a subwavelength electrostrictive interface that induces up to a twofold local change in wave velocity. We demonstrate diffraction-limited image formation with a focal spot of half the source wavelength, tunable imaging at complementary frequencies, and control of object-image relations using curved interfaces.

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1. Introduction

A rapidly expanding frontier in metamaterials has emerged from the recognition that the time dimension offers a rich and largely unexplored axis for wave control. In time-varying media, whose electromagnetic, acoustic, or mechanical properties are modulated in time rather than, or in addition to, space, temporal modulation conserves momentum while allowing energy exchange with the medium, giving rise to frequency conversion, amplification, temporal reflection, and momentum bandgaps [1–7]. A comprehensive framework now exists for the photonics of time-varying media [1], with related developments in Floquet metamaterials [2], photonic time crystals [5,8,9], and spacetime metamaterials [10,11]. This vision of time as an engineering degree of freedom owes much to the pioneering insight of Nader Engheta, who recognized early that temporal modulation could dramatically enlarge the design space of metamaterials and wave–matter interactions [12,13].

Within this landscape, active spatiotemporal surfaces occupy a central position: by combining spatial confinement with temporal modulation, they reach functionalities beyond purely spatial or purely temporal transformations alone. Demonstrated effects include temporal parity-time symmetry [14], nonreciprocal amplification [15], Fresnel drag [16], parity-time-symmetric negative refraction [17], Archimedes-screw transport of light [18], a range of wavefront-control strategies [19–23], and space-time-coded digital metasurfaces that jointly steer propagation direction and harmonic content [24,25]; the family has recently expanded to phase conjugation and spin effects at space-time interfaces [26]. Two complementary routes have emerged. The first uses temporally varying reflective surfaces, which established time reflection and refraction as basic wave processes [27] and enabled temporal reflection, frequency conversion, and time diffraction [3,4], second-harmonic generation at a time interface [28], continuous-time-crystal behavior [29], and quantum time reflection of ultracold atoms [30]. The second route, central to this work, relies on transmission through a thin active layer: once its thickness is subwavelength, the

out-of-plane momentum constraint is lifted, allowing negative refraction and phase conjugation in the symmetric half-space and turning the layer into a flat time-reversal (TR) lens [31,32].

Realizing such transmitting surfaces is demanding: the modulation must be simultaneously large and fast compared with the wave period, yet confined to subwavelength dimensions. In optics this has recently become possible with nonlinear plasmonic films, graphene, and epsilon-near-zero media [33–37], which support strongly enhanced nonlinear and time-dependent responses [38]. These platforms have convincingly produced phase-conjugated and negatively refracted transmitted beams, validating the key ingredients of flat TR lenses [31,32]. Yet they stop at the conjugated beam itself: using such interfaces as extended elements that focus or reconstruct arbitrary wavefronts, as genuine imaging devices, has remained essentially unexplored experimentally.

Water waves offer an ideal setting to close this gap. Large effective-index modulations can be produced on timescales shorter than the wave period — by vertically shaking the bath or through electrostriction — enabling instantaneous time mirrors [39], phase-conjugate mirrors driven by parametric instability [40], disordered time-varying media [41], spatiotemporal frequency cascades [42], and broadband refractive control via electric-field modulation of the wave speed [43]. Here we show that a localized electrostriction drive produces strong, subwavelength refractive-index modulations, well suited to thin active boundaries such as flat TR lenses [31,32]. Using this drive, we realize a water-wave TR lens that forms images at half the modulation frequency; we show that it can also be operated at detuned frequencies, creating an imaged field at the complementary frequency; and we demonstrate that bending the lens controls the object–image relation.

2. Principle and implementation of a flat TR lens for water waves

Figures 1(a) and (b) show the principle of an active meta-surface to realize a flat TR lens by time-modulating the refractive index in a uniform layer of sub-wavelength thickness [31,32]. A thick layer of such a material would act as a standard phase conjugation (PC) mirror that creates a time-reversed version of the input field. The field from a point source would thus re-focus at the source (Fig. 1(a)). By reducing the thickness of the layer so that its effective dimensionality is lowered, one lifts out-of-plane momentum conservation, allowing a mirrored copy of the time-reversed field to form and focus on the other side of the film (Fig. 1(b)). The active meta-surface thus acts as a flat TR lens.

It was recently pointed out that for water waves a strong and fast time-modulation of the wave celerity can be achieved by applying a vertical acceleration jolt to the liquid, creating an efficient TR mirror [39]. With a sinusoidal vertical acceleration, a PC mirror for water waves was produced [40]. However, implementing a TR lens requires a narrow spatial confinement of the modulation, which cannot be achieved with a global mechanical excitation.

The presence of a uniform external electric field modifies the standard gravity-capillary wave dispersion relation, $\omega_0(k)$, to $\omega(k)^2 = \omega_0(k)^2 - \epsilon E^2 k^2 / (\rho \tanh(kD))$ [44–46], where ϵ denotes the electric permittivity of air, E is the electric field strength at the water surface, and D is the electrode-to-free-surface distance (see Supplement 1). To concentrate the effect along a single line, we suspended a thin high-voltage line a few millimeters above the water surface (Fig. 1(c)). We applied electric field strengths between 0 and 20 kV/cm, corresponding to a twofold change in wave celerity. A localized monochromatic point source of angular frequency $\omega_s = 2\pi f_s$ was generated by directing a narrow air jet onto the free surface. A continuous positive airflow provided a steady pressure bias, while a loudspeaker modulated the airflow at the prescribed source frequency, producing a stable and approximately sinusoidal forcing of the water surface (see Fig. S1). The full space- and time-resolved wave field around the TR lens was measured using a digital Schlieren method [49]. This allowed us to reveal the waves produced by the active

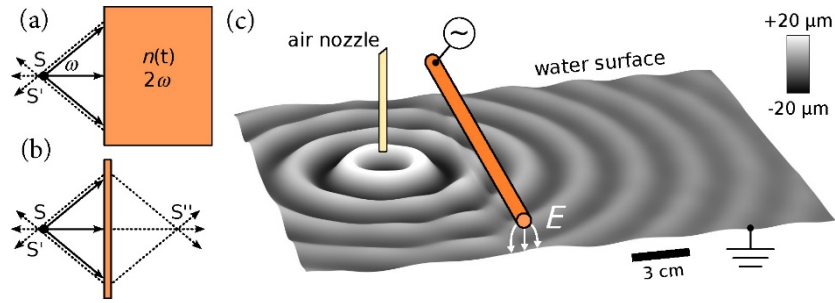


Fig. 1. Concept and experimental implementation of a time-reversal (TR) lens for water waves. (a) Standard phase-conjugation mirror. Waves emerging from a point source S at frequency ω propagate into a region of space where the refractive index is modulated at frequency 2ω . Frequency mixing produces a conjugated wave field S' that propagates in a time-reversed manner and focuses back at the source. (b) A thin PC-mirror acts as a TR lens: the same sources which created S' now produce a forward focused copy S'' . (c) TR lens setup for water waves. A thin rod-shaped electrode is placed above the water surface. An AC voltage applied to the rod induces a locally modulated electric stress at the water surface. A point source is created by blowing on the water surface. A surface plot of the measured input wave field is shown.

lens by subtracting from each recorded field a reference recording in which the lens was kept inactive.

The electric line acts as a TR lens if its modulation frequency ω_L is set to $2\omega_s$. Besides plane waves at frequency $2\omega_s$ formed directly (independent of the input field) by the time-varying electric stress below the line, frequency mixing between the input field and the active line generates waves at the sum and difference frequencies $3\omega_s$ and $-\omega_s$ (see [Supplement 1](#)). The $-\omega_s$ component corresponds to the backward focused time-reversed field S' and its forward focused counterpart S'' , while the $3\omega_s$ component represents diverging waves of a smaller wavelength. In conventional PC experiments (Fig. 1(a)) the $3\omega_s$ component would be very weak because of the phase matching condition imposed on both parallel ($k_{\parallel}$) and perpendicular ($k_{\perp}$) wave vector components with respect to the interface. For a sufficiently thin PC region (Fig. 1(b)) there is no restriction on $k_{\perp}$. Besides allowing for the formation of a forward focused wave with $|\omega| = \omega_s$, this also facilitates the generation of waves for which $|\omega| \neq \omega_s$. As we will see below, this gives some flexibility in tuning the focal distance of the lens.

Interestingly, the same relaxation of the phase-matching condition that enables the lens to refocus on the opposite side of the interface also removes the usual bulk phase-matching restriction associated with dispersion. Because the PC region is deeply subwavelength, no significant phase mismatch can accumulate across the interaction region and the usual bulk phase-matching condition of an extended nonlinear medium is therefore absent. Instead, the generated wave inherits the spatial phase distribution of the incident field along the interface. At exact resonance, the phase-conjugated wave has the same absolute frequency and therefore the same wavenumber magnitude as the incident wave, so that dispersion does not modify the mirror-symmetric imaging relation. TR lenses can consequently also operate in strongly dispersive media, such as gravity-capillary water waves, under detuned conditions. In this case, dispersion determines the wavenumber of the converted wave and, together with the phase distribution imposed at the interface, sets its propagation geometry and hence the focal distance.

3. Characterization of a flat TR lens for water waves

An example of a measured wave field produced by the TR lens is shown in Fig. 2(a) (see also [Visualization 1](#) and [Visualization 2](#)). Here, the source was operated at $f_s = 8$ Hz and the lens frequency was matched at $f_L = 2f_s = 16$ Hz. To single out the time-reversed part which is of interest here, we filtered the recording at $-\omega_s$ in the frequency domain. Both the time-reversed wave and its forward copy are clearly visible. The conversion efficiency η , defined as the ratio a_s''/a_s between the amplitude a_s'' of the forward focused field and the amplitude a_s of the input field, was about $\eta = 5\%$ in this case. This is comparable to what can be achieved in state-of-the-art optical 4WM experiments that rely on intense pumping of thin film materials [33,34,36]. However, in our experiment there is no trade-off between gain density and the extension of the active region. To increase the conversion efficiency, we can simply increase the voltage supplied to the electric line (as long as discharge can be avoided). At high electric field amplitudes, we observed the formation of a Faraday instability in the form of standing waves localized along the length of the line and growing exponentially in time [47,48], evidencing the extreme local parametric amplification that can be achieved in the system without the need for a cavity. A good measure of this crossover is the forward-to-backward phase-conjugated amplitude ratio $\xi = a_s''/a_s'$, shown in Fig. 2(b).

As shown analytically and numerically in the [Supplement 1](#), the electrode-to-surface distance D sets the transverse width of the electric-stress profile and therefore the effective thickness of the active PC region. Decreasing $k_s D$ thus drives the system from a bulk-like PC regime toward the thin-interface limit. At $k_s D \sim 1$, corresponding to $D \simeq \lambda_s/2\pi$, the interaction region is deeply subwavelength and the forward and backward phase-conjugated components become nearly symmetric. For larger $k_s D$, phase mismatch can progressively accumulate across the active region and the forward component is suppressed.

From the complex wave amplitude $a(x, y)$ obtained in the frequency domain we computed the intensity $|a|^2$. The focal position was determined from the position of the intensity maximum, and the FWHM from the transverse intensity profile through the focal region. The numerical uncertainty associated with locating the maximum and half-maximum positions is smaller than the marker size. The dominant uncertainty instead arises from the non-ideal and slightly asymmetric structure of the experimental focal field, as visible in Fig. 2(c) and in the Supplementary Visualizations. From the typical variation of the focal profile across the focal region, we estimate uncertainties of approximately $\pm 0.2\lambda_s$ for the focal position and $\pm 0.1\lambda_s$ for the FWHM. As shown in Fig. 2(c) we observe a sharp focal spot behind the lens. The full width at half maximum (FWHM) of the spot is roughly half the source wavelength λ_s , indicating that the imaging resolution is diffraction limited (see [Supplement 1](#)). Unlike a normal lens, a TR lens is expected to have a focal plane which moves in a mirror-symmetric way with respect to the object plane. We confirmed this by scanning the object distance d_s while measuring the focal distance d_s'' (Fig. 2(d)). For each source frequency, the lens modulation frequency was adjusted to maintain the resonant condition $f_L = 2f_s$, hence $f_L = 14, 16$ and 18 Hz for $f_s = 7, 8$, and 9 Hz, respectively. A slight increase in spot width with object distance was observed (Fig. 2(e)), consistent with a decrease in the numerical aperture with distance (dashed line, Fig. 2(e)).

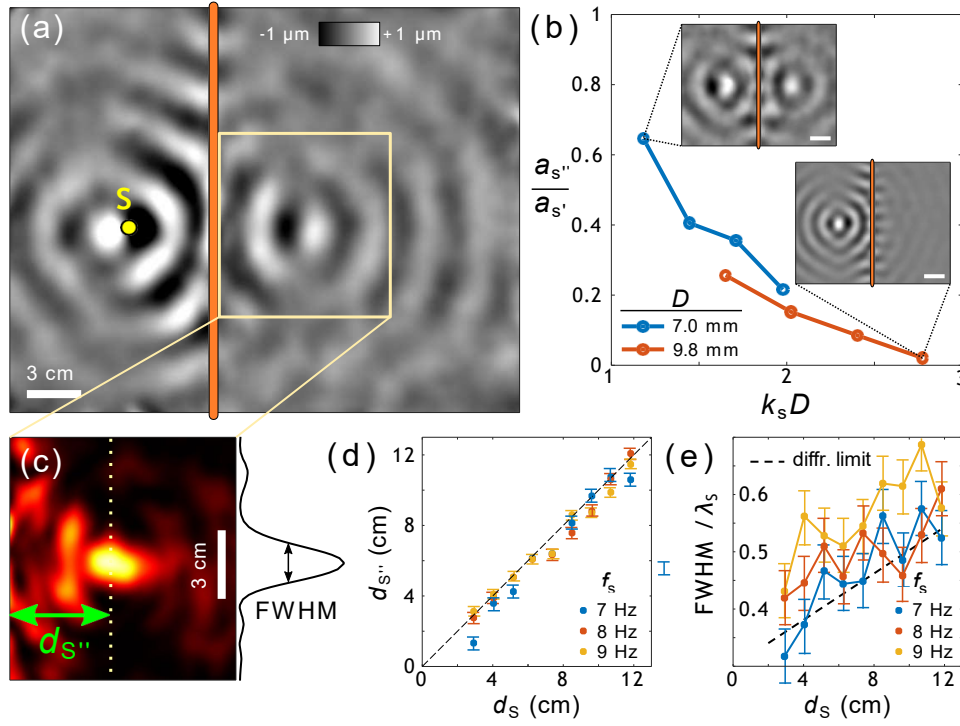


Fig. 2. Characterization of the water wave TR lens. (a) Measured wave field. The source (yellow dot) was operated at 8 Hz and the electric line is modulated at 16 Hz. The field is filtered at -8 Hz in the frequency domain. (b) Forward/backward amplitude ratio $\xi = a_{s''}/a_{s'}$ as a function of the dimensionless lens width $k_s D$. For a given D , each point is obtained by exciting at 7 Hz, 8 Hz, 9 Hz and 10 Hz (from left to right). Wave fields for the extreme cases are shown. (c) The focal distance $d_{s''}$ and the spot FWHM are extracted from the wave field intensity. (d) Focal distance versus source distance d_s for various source frequencies f_s (with $f_L = 2f_s$). The dashed line indicates the expected mirror symmetry. (e) Dependence of FWHM (normalized by source wavelength λ_s) on d_s . The dashed line indicates the theoretical diffraction limit. Error bars indicate the typical uncertainty associated with the non-ideal spatial structure of the experimental focal spot rather than statistical fluctuations between repeated measurements. The applied voltage is 7 kV modulated with ± 1 kV.

4. Frequency shift TR lenses

An interesting consequence of the absence of the usual bulk phase-matching constraint is that the lens can also operate under detuned conditions ($\omega_L \neq 2\omega_s$), even though the converted wave then has a different frequency and wavenumber from the incident wave. In Fig. 3(a) we show a wave field obtained with $f_L = 2f_s + 1$ Hz ($f_s = 8$ Hz) (see Visualization 3). In this case there was no need to subtract the source wave field in our analysis as the time-reversed waves now appear at $|f| = 9$ Hz, one Hertz above the source frequency. As is seen in Fig. 3(a) these waves of reduced wavelength still form a clear focal spot, but at a larger distance from the lens. This effect can be understood by considering the phase matching condition that still holds along the length of the thin lens. In the Fresnel approximation, the phase of the source wave field here varies as $k_s x^2 / (2d_s)$. Phase-matching thus requires $k_s''/d_s'' = k_s'/d_s' = k_s/d_s$, implying that the focal distances d_s' and d_s'' are increased by a factor k_s''/k_s . Similarly, when the line is operated below resonance the focal distance decreases by this factor (Fig. 3(b) and Visualization 4). The

predicted change in slope between d_s'' and d_s is indeed observed experimentally (Fig. 3(c)). For the off-resonance conditions presented here ($\Delta f = \pm 1$ Hz), the converted-wave amplitudes remain comparable to the resonant case over this moderate detuning range.

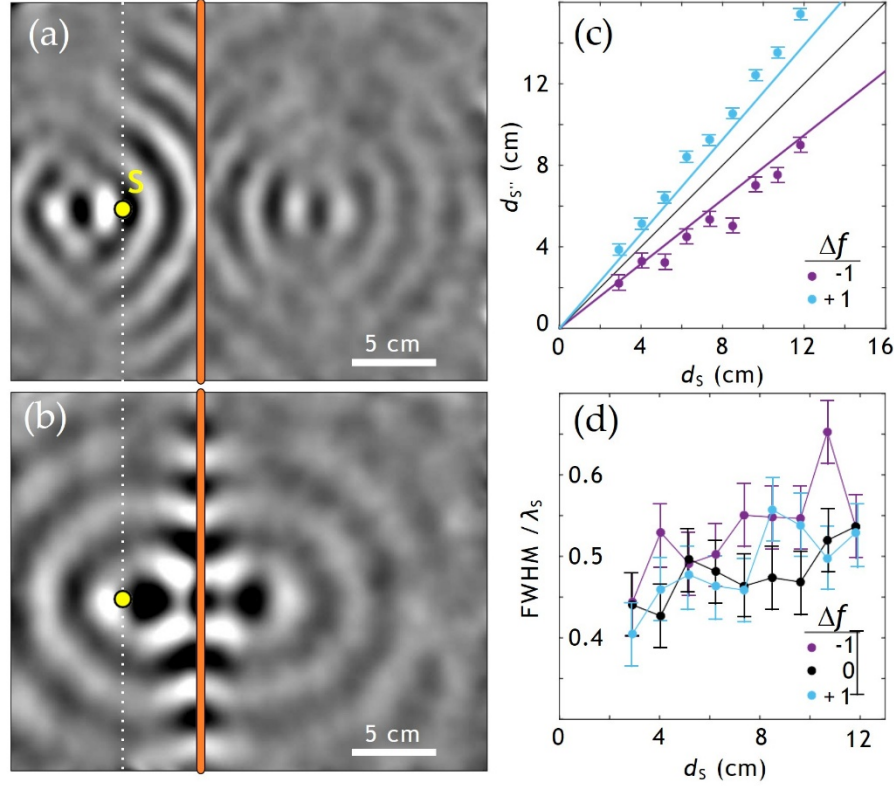


Fig. 3. TR lens operating at off-resonance conditions $f_L = 2f_s + \Delta f$. (a) Measured wave field for $f_s = 8$ Hz and $\Delta f = +1$ Hz (filtered at 9 Hz). (b) Measured wave field for $f_s = 8$ Hz and $\Delta f = -1$ Hz (filtered at 7 Hz). (c) Focal distance d_s'' versus source distance d_s for $\Delta f = \pm 1$ and $f_s = 8$ Hz. Solid lines indicate the paraxial prediction. (d) Corresponding spot FWHM. Error bars indicate the typical uncertainty associated with the non-ideal spatial structure of the experimental focal spot rather than statistical fluctuations between repeated measurements. 11 kV modulated with ± 3.5 kV.

Interestingly, for these moderate off-resonance conditions the width of the focal spot is the same as in the resonant case (Fig. 3(d)). This can be directly understood from the fact that even under off-resonant conditions the generated wave inherits the $k_{\parallel}$ spectrum of the source. Going beyond the Fresnel approximation, one finds that the opening angle Θ over which the phase of the circular input wave fronts can be matched along the line scales as $\Theta \sim (d_s |k_s - k_s''|)^{-1/4}$, so that for larger detuning a decrease in resolution can be anticipated.

5. Curved TR lenses

Although the TR lens acts like a lens in the sense that an image is formed on the far side of it, the transformation it performs has much in common with that of a mirror. A hint of this relationship is already apparent from the fact that the image plane of a flat TR lens moves in a mirror-symmetric way with respect to the object plane (*cf.* Figure 2(d)). This mirror analogy can be made explicit in the geometrical-wave limit. Decomposing the incident wavevector locally as $\mathbf{k} = \mathbf{k}_{\parallel} + \mathbf{k}_{\perp}$ a

conventional mirror produces $\mathbf{k}_{\text{refl}} = \mathbf{k}_{\parallel} - \mathbf{k}_{\perp}$ whereas the forward phase-conjugated branch of a resonant thin TR lens produces $\mathbf{k}_{\text{TR}} = -\mathbf{k}_{\parallel} + \mathbf{k}_{\perp} = -\mathbf{k}_{\text{refl}}$. The TR-lens ray therefore follows the same geometrical path as the corresponding mirror ray but in the opposite direction, thereby interchanging real and virtual images. This correspondence holds in the local geometrical-wave limit for a sufficiently thin interface and resonant conversion between waves of equal wavenumber magnitude. For example, a parabolic-shaped TR lens would collimate waves emerging from a point source placed at its focus, with the collimated beam emerging on the far side of it (Fig. 4(b)). An experimental demonstration of this phenomenon is shown in Fig. 4(c) (see Visualization 5), where we replaced the straight electrode by a curved one. Note that this property of the forward projected wave field is markedly different from that of the conventional time-reversed field, for which the back propagation is independent of the shape of the active region [40]. The parabolic geometry used here is designed for a source located at its focus. If the source is displaced, the incident phase distribution along the interface changes and the forward phase-conjugated field is no longer perfectly collimated. In the geometrical-wave limit, its evolution follows the reversed-ray construction of the corresponding parabolic mirror: a small transverse displacement mainly changes the output direction, with increasing aberrations away from the paraxial regime, whereas an axial displacement produces a converging or diverging output. This sensitivity to source position concerns the forward field S'' ; the backward time-reversed component S' continues to retrace the incident field toward the source.

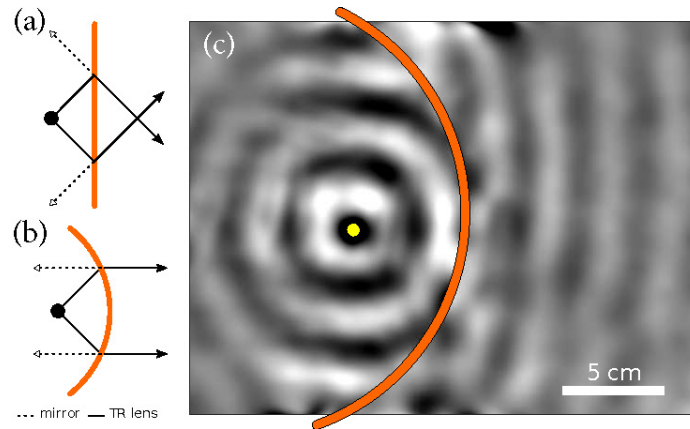


Fig. 4. Wave field produced by a curved TR lens. Theoretical ray paths for (a) a flat TR lens and (b) a curved TR lens (solid lines with arrows). Dashed lines show the ray paths for a mirror. In both cases the TR lens forms a real image at the virtual image location of an equally shaped mirror surface. (c) Measured wave field for a parabolic-shaped TR lens. The source is placed at the focus of the parabola. The applied voltage is 7kV modulated with ± 1 kV.

6. Conclusion

Beyond the specific TR lens geometry demonstrated here, our results establish water waves as a versatile testbed for the broader physics of active spatiotemporal boundaries that has emerged across photonics, acoustics, and electromagnetics over the past several years [1,2,8,9,12,13]. The platform is naturally suited to probing several of the open questions currently driving this field: the role of momentum bandgaps and parametric amplification in extended, spatially confined geometries [5,8,15]; the interplay between nonreciprocity and spatial localization in space-time-modulated interfaces [6,7,24,25]; and the experimental study of wave propagation in

media with arbitrarily moving, sub- or super-luminal refractive index profiles [10,11,16]. More generally, we anticipate that the simplicity and tunability of electrically actuated water-wave interfaces will make them a useful complement to photonic and acoustic platforms for exploring the rapidly expanding landscape of four-dimensional wave control [12,13].

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

Supplemental document. See [Supplement 1](#) for supporting content.

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